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Evolution of Tenacity in Mixed Mode Fracture – Volumetric Approach

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In fracture mechanics most interest is focused on stress intensity factors, which describe the singular stress field ahead of a crack tip and govern fracture of a specimen when a critical stress intensity factor is reached. In this paper, stress intensity factors which represents fracture toughness of material, caused by a notch in a volumetric approach has been examined, taking into account the specific conditions of loading by examining various U-notched circular ring specimens, with various geometries and boundary conditions, under a mixed mode I+II. The bend specimens are computed by finite element method (FEM) and the local stress distribution was calculated by the Abaqus/CAE. The results are assessed to determine the evolution of the stress intensity factor of different notches and loading distances from the root of notch. This study shows that the tenacity is not intrinsic to the material for all different geometries notches.

Keywords: equivalent notch stress intensity factors, mixed mode fracture, stress intensity factor, tenacity, volumetric approach.

1. Introduction

The analysis of the stress distribution and the calculation of stress intensity factor to a U-notched specimen have been widely studied with many different methods. But less attention has been paid to the study of a mixed mode fracture. More than 60 years ago, the stress intensity factor SIF for an infinite plane has been studied by Irwin [8]. The SIF describes the stress state at a crack tip, is related to the rate of crack growth, and is used to establish failure criteria due to fracture. The stress intensity concept is based on the parameter K, which quantifies the stresses at a crack tip. The concept of the critical stress intensity factor has a natural application in the case of the determination of fracture toughness of very brittle materials such as steel. With this material, it is very difficult to prepare precrack samples to measure the fracture toughness, because the precracking operation by static loading leads frequently to the undesirable fracture of the material, so notched specimens are generally used to determine the fracture toughness. Generally, evaluation of this mechanical characteristic is done by using the linear elastic fracture mechanism and considering the notch as a crack. The fracture toughness K_I is related to the critical load by the following relationship [2]:

$$\sigma_{ij} = \frac{K_I}{\sqrt{2\pi r}} f_{ij}(\theta) + C_1 r_0 + C_2 r_{1/2} + \ldots \tag{1}$$

If r is very small, the first term of the solution is very large (infinite for $r = 0$); therefore, the other terms can be neglected. Because all cracking and factoring take place at or very near the crack tip (where $r \geq 0$), it is justifiable to use only the first term of the solution to describe the stress field in the area of interest. For the stress in the x-direction along the plane $\theta_0 = 0$, the function $f_{xx}(\theta_0) = 1$, so that:

$$\sigma_{ij} = \frac{K_I}{\sqrt{2\pi r}} \text{ for } \theta_0 = 0\,. \tag{2}$$

However, methodology for determination of fracture toughness in mode I and II [2, 5, 8, 10] is well established and since a long time has been given to crack initiation. Since on the one hand, for an elastoplastic ductile material, any crack tip or notch tip will be blunted under loading due to the fracture process zone (volumetric approach), and then lose its singularity from a blunt notch specimen and crack initiation appears preferentially on stress concentration. For that, it seems very interesting to us to obtain first the stress distribution in the root of a notch and then studied the evolution of the tenacity. In this paper we will present a general analysis on the evolution of tenacity for a ductile material in elastoplastic mode. A new parameter K_ρ, the stress intensity factor defined for blunt notch is determined from the calculated stress distribution around the notch. The influence of the angle and acuity of notch on the critical notch stress intensity factor $K_{\rho c}$ is studied.

2. Materials and methodology

2.1. *Material*

The material studied is a high strength steel S1463-45CrMoSi6 according to French standard. Mechanical properties are listed in Tab. 1, where: ν, E, σ_Y, σ_U, A [%], and K_{IC} are Poisson's ratio, modulus of elasticity, yield stress, ultimate stress, relative elongation and fracture toughness respectively. Microanalysis of the material gives the following chemical composition: 0.45%C, 1.6%Si, 0.6%Mn, 0.6%Cr, and 0.25%Mo.

Table 1 Mechanical properties of 45CDS6 Steel

E [MPa]	ϑ	σ_Y[MPa]	σ_U [MPa]	A [%]	density [kg/m^3]	K_{IC} [MPa]
210065	0.28	1463	1662	2.8	7800	97

2.2. *Specimens*

The experiments have carried out by examining various U-notched circular ring specimens, with various geometries and boundary conditions (Fig. 1), [4,5], where: external radius $R_e = 20$ mm, internal radius $R_i = 10$ mm, thickness $B = 7$ mm, and notch length $a = 4$ mm. Different notch radii are introduced using a wire-cutting electrical discharge machine and using wires of different diameter. The notch root radius was measured using a profile projector. Five notch radius values are used: $\rho = \{0.15, 0.3, 0.5, 1, 2\}$ mm. With: $0° < \alpha < 33°$ (mixed mode I+II), [5].

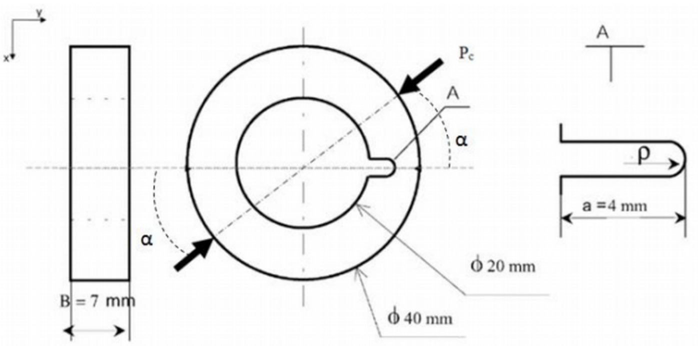

Figure 1 U-notched circular ring specimen

The specimens are submitted to compression load (Fig. 2) in order to determine the critical loads when the fracture occurs. These loads are introducing to the simulation computation to finally evaluate the stress triaxiality evolution.

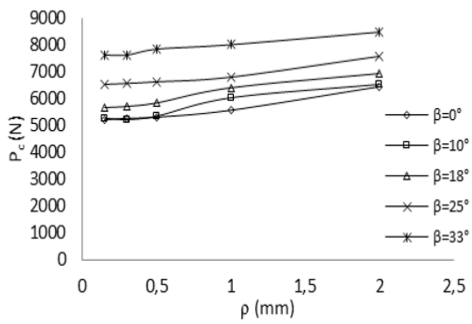

Figure 2 Critical load P_C vs. ρ for various inclination angles β

3. Finite element analysis and stress distribution

The nonlinear finite element simulations are performed using ABAQUS 6.10. The geometry of the U-notched circular was simplified by considering a plane part worthless thickness such as $e = 1$.

To study the transitional stages of mode I to the mode II, it's necessary to define new orientations of stress. There is an angle corresponding to each mode of application of load named θ_0 [10]. Method XFEM is very effective to study this kind of problem (applicable on Abaqus/CAE). The following figure (Figs. 4 and 5) summarizes the values of the angles drawn on (Abaqus/CAE) and raised.

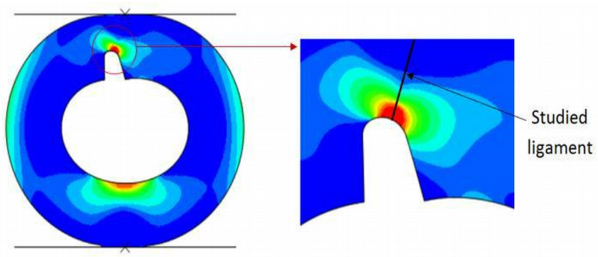

Figure 3 Studied ligament

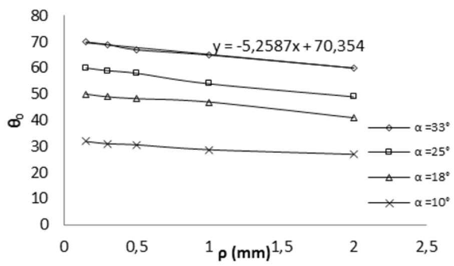

Figure 4 Evolution of fracture angle θ_0 vs. radii ρ

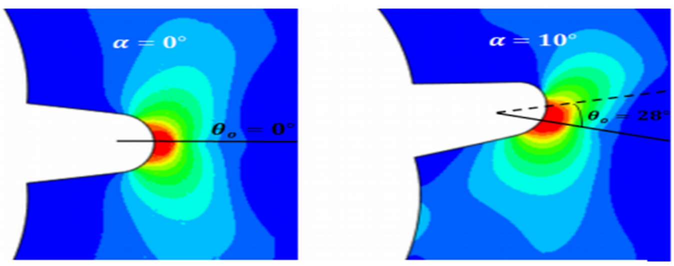

Figure 5 Evolution of fracture angle θ_0

The numerical values obtained according to θ are compared to the experimental values measured by an optical microscope [4]. In mode II (for $\alpha = 33°$), for a radius notch $\rho = 0$: $\theta_0 = 70.35°$.

This value is compared to others results, such as: $\theta_0 \approx 70.39°$ [4,5] and $\approx 70.5°$ [7], and $\approx 70.33°$ [9]. The evolutions of stress with various radii are presented in the sections at the bottom, in mixed-mode (I+II) fracture crack initiation from notches is governed by the tangential stress. The stress evolutions are plotted versus the notch tip distance for each angle α, for ($\rho = 1$), Fig. 6. The maximum stress values decrease when the notch radius increases [11].

According to these results, we notice that the maximum stress on the ligament is always that of opening one with a remarkable increase in shear stress, because of shearing of the lips of the notch. Furthermore, we found a small difference between the maximum stress σ_{xx} and σ_{vm}. We assume that the mixed-mode (I+II).

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4. Effective notch stress intensity factor and volumetric method

Stress distributions around the notch defect have been converted into so called notch stress intensity factor using the notch fracture mechanics and particularly the volumetric method. The volumetric method is a local fracture criterion, which supposes that the fracture process requires a certain fracture volume. This volume is assumed as a cylinder [?] with effective distance at its diameter. The elastic-plastic stress distribution along the ligament is plotted in the logarithmic diagram as can be seen in Fig. 8.

Two distinct zones in the diagram can be distinguished:

- Zone I: the elastic-plastic stress drops gradually in the elastic regime.
- Zone II: starts at a certain distance which is named the effective distance. It represents linear behavior in the bi-logarithmic diagram.

The notch stress intensity factor is defined as the function of effective distance and effective stress:

$$K_\rho = \sigma_{\text{eff}} \sqrt{2\pi X_{\text{eff}}} \tag{3}$$

where: X_{eff} and σ_{eff}, are effective distance and effective stress.

By definition, the effective distance is the diameter of the process volume assuming it has a cylindrical shape. To determine this effective distance studied the evolution of the function of the gradient relative of stress to the bottom of notch. This function represents a minimum corresponding to the effective distance X_{eff}:

$$(\mathrm{r}) = \frac{1}{\sigma_{\mathrm{xx}}(\mathrm{r})} \frac{\mathrm{d}\sigma_{\mathrm{xx}}(\mathrm{r})}{\mathrm{dr}} \tag{4}$$

where: r and $\sigma_{\mathrm{xx}} r$, are relative stress gradient and maximum principal stresses or crack opening stress, respectively.

As we have assumed previously, the effective distance is the diameter of the process volume, assuming it has a cylindrical shape. The process zone distance is relatively small and strongly connected with the microstructure (grain size or other metallurgical characteristic). In this case the effective distance is considered as equal to the microstructure characteristic. In volumetric method, however is a local fracture criterion, the effective distance is not a material constant but a characteristic of the stress distribution which depends on geometry, loading mode, load level etc. [11]. However stresses are multiplied by a weight function in order to take into account the influence of stress gradient due to geometry and loading mode. The effective stress is defined as

$$\sigma_{\mathrm{eff}} = \frac{1}{X_{\mathrm{eff}}} \int_0^{X_{\mathrm{eff}}} \sigma_{xx}(r)\,(1 - r\chi(r))\,\mathrm{d}r \tag{5}$$

5. Critical equivalent notch stress intensity factor

By assumption, we consider that a rupture in mixed mode is controlled by a criterion with two parameters, a principle effective stress and an effective distance. This assumption rises owing to the fact that the crack mechanism occurs when the average stress in the volume of development of the process of rupture exceeds a breaking value. The volume of development of the process of rupture is supposed to be cylindrical of diameter X_{eff}. In the case of a notch under mixed mode I+II, the stress intensity factor equivalent is written then [3]:

$$K_\rho = \sigma_{\mathrm{eff}}\sqrt{2\pi X_{\mathrm{eff}}} \tag{6}$$

Under the critical conditions and according to the direction of starting $\theta = \theta_0$ [11]:

$$K_{I\rho}^{C} = K_{\mathrm{eq}\rho}^{C} = \sigma_{\mathrm{eff}}^{C}\sqrt{\pi X_{\mathrm{eff}}^{C}}\,, \tag{7}$$

where: $K_{I\rho^C}$ is the critical stress intensity factor, is the critical effective stress and critical effective distance.

Equation (7) is defined as the expression of a proposed critical notch stress intensity factor which represents the fracture toughness of the materials and is a function of the maximum stress at the notch tip and the characteristic distance. The units are [MPa $\sqrt{m}$].

6. Results and discussion

As it is already treated, the definition of the stress intensity factor K according to the mode of request is presented by the equation 7. In the table below, a presentation of the distribution of the notch stress intensity factor $K_{\rho\alpha}^{c}$, and influences it rays of the notches on this last. It summarizes the various values of the NSIF in different mode (I, I+II and II).

Table 2 gives values of $K_{\rho\ \alpha}^{c}$ provided by Eq. (7) at very different distances ρ from the notch tip. It can be found that the critical value of $K_{\rho\ \alpha}^{c}$ increases with an increase of the notch radius. So this criterion is inconvenient to determine the material intrinsic fracture toughness. Only the fracture toughness $K_{\rho\alpha}^{c}$ is dependent of ρ and α and is really an intrinsic fracture toughness parameter.

The evolution remained stable throughout the U-notched specimens, and while converging towards a value critic near to 97 MPa $\sqrt{m}$ [5]. An interesting relationship can be set up between the stress concentration factor and the critical notch stress

Table 2

ρ [mm]	Mode I	Mode I+II			Mode II
	$K^C_{I\rho\,\alpha=0°}$	$K^C_{\rho\ \alpha=10°}$	$K^C_{\rho\ \alpha=18°}$	$K^C_{\rho\ \alpha=25°}$	$K^C_{II\ \rho\ \alpha=33°}$
0.15	98.89	95.5	96.25	89.41	71.17
0.3	98.89	95.47	101.15	92.86	78.93
0.5	92.83	95.62	95.01	109.16	91.56
1	94.27	108.85	117.22	121.49	114.85
2	100.06	106.78	132.24	143.66	156.41

intensity factor $K^{\alpha}_{\rho c}$ through the medium of the significant distance X_{eff}. Figures 9(a), (b) and (c) show the influence of ρ and angle of orientation α on the critical notch stress intensity factor $K^{\alpha}_{\rho c}$. It is noted that the evolution of these factors is in growth according to α, more the angle of orientation of the ring increases, more the $K^{\alpha}_{\rho c}$ factor loses its stability. Furthermore, the critical notch stress intensity factor distribution at the notch tip can be schematically represented as a zone where the CNSIF is quasi-constant over the studied ligament for:

$$0 \leq \rho \leq \rho_c \rightarrow \lim_{\rho \to \rho_c} K^c_{\rho\alpha} = K_{IC} = 97\,\text{MPa}\sqrt{m}$$

Once the radius ρ exceeds a value of $\rho > \rho_c$, the factor $K\rho c\alpha$ increases strongly. And between the two phases of transition exists a value of radius critic ρ_c, a value of 0.75 mm, beyond this value a light increase with the radius of notch is mentioned. This value was mentioned by others authors [5]. Generally, fracture toughness is considered as identical in mode I and mode II. However some seldom experimental results indicate a small difference between these fracture toughness. We considered also that this difference exists and define a “fracture mode resistance ratio” ζ:

$$\zeta = \frac{K_{\text{II c}}}{K_{\text{I C}}}\,. \tag{8}$$

Figure 10 illustrates the distribution of the ζ report according to the radius of notch, with $\zeta = 0.70$. The value of the ζ report is practically given by several authors [5,6,8], which is equal to ≈ 0.86 for steel material [7].

7. Conclusion

The results are assessed to determine the evolution of the stress intensity factor of different notches and loading distances from the root of notch. This study shows that the tenacity is not intrinsic to the material for all geometry structure for:

$$\rho \leq \rho_c \rightarrow \lim_{\rho \to \rho_c} K^c_{\rho\alpha} = K_{IC}\,.$$

In determining the stress intensity factors, the effect of the loading distance and the crack distance from the middle of specimen should be considered and for this purpose fivefactors should be used. The entire distribution needs five parameters to be described: the stress concentration factor K_{IC}, the effective stress σ_{eff}, the effective distance X_{eff}, defect orientation α and the critical notch stress intensity factor $K_{c\rho}$. The critical stress intensity factor for a blunt notch can be defined by the following relationship:

$$K_{\rho}^{c}\left(\rho \leq \rho_{c}\right)=\sigma_{\text{eff}} \sqrt{2 \pi X_{\text{eff}}}\,.$$

This quantity represents the fracture toughness of a brittle material and can be measured on crack or notch. It is independent of the notch radius and consequently intrinsic to this material. The standardisation of the technique is still in progress and is yet to be fully verified so far. The results are encouraging ever for multi-axial loading conditions

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